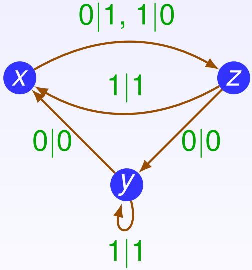
On the finiteness and the order problems for automaton (semi)groups

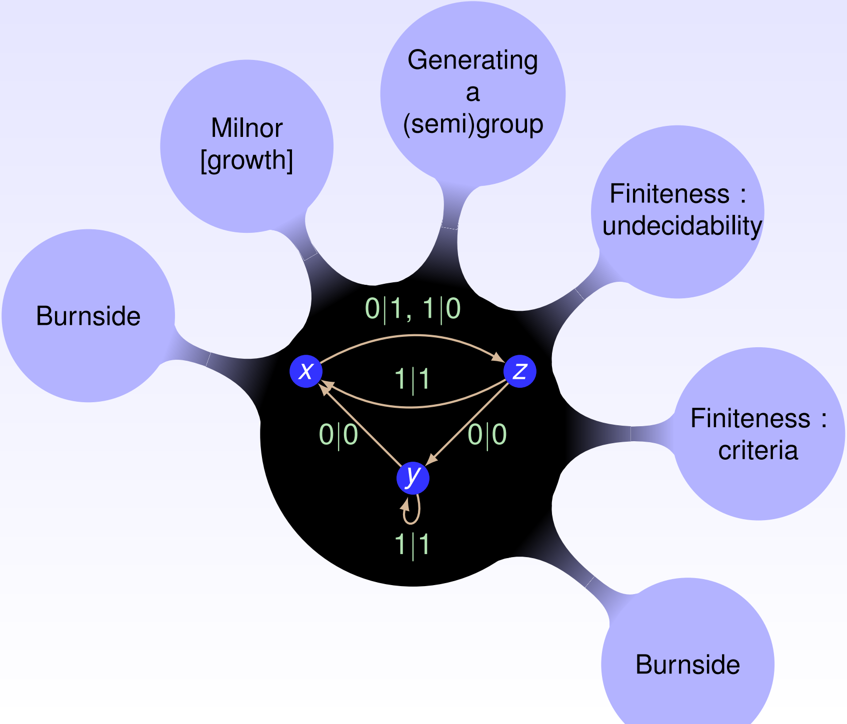
Ines Klimann

LIAFA – UMR 7089 CNRS & Université Paris Diderot

MealyM
JCJC-12-JS02-012-01

GT sda2


0|1, 1|0
x
1|1
z
0|0
0|0
y
1|1


Generating a (semi)group
Milnor [growth]
Finiteness : undecidability
Burnside
Finiteness : criteria
Burnside
0|1, 1|0
x
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1|1
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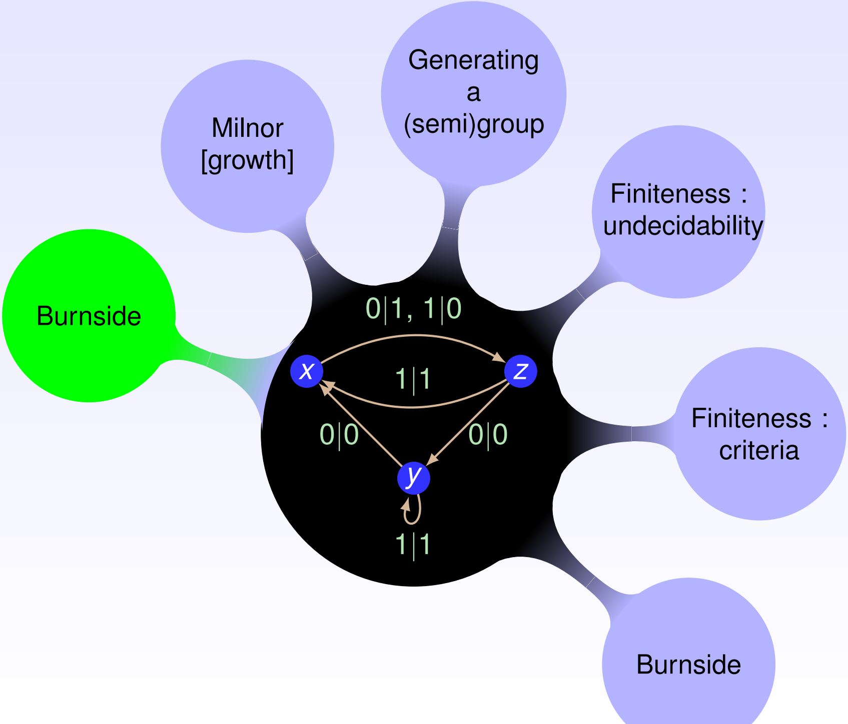
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x
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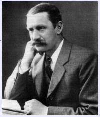
y

1902

Is a finitely generated group whose all elements have finite order necessarily finite ?



Burnside

1902

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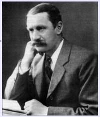
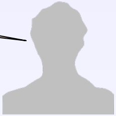


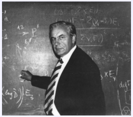
Burnside

1964

No!



Golod



Shafarevich

1902

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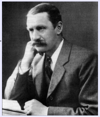

Burnside

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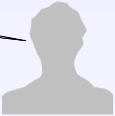
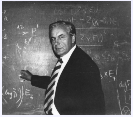
No!



Golod

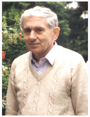


Shafarevich

1968

Even if the orders are bounded.

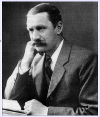
Novikov



Adian

1902

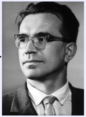
Is a finitely generated group whose all elements have finite order necessarily finite ?



Burnside

1961

With automaton groups ?



Glushkov

1902

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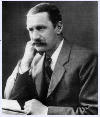
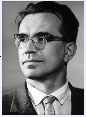


Burnside

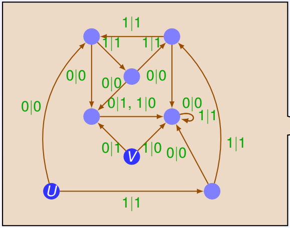
1961

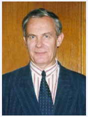
With automaton groups ?



Glushkov

1972





Aleshin

1902

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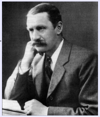
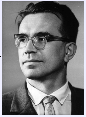


Burnside

1961

With automaton groups ?



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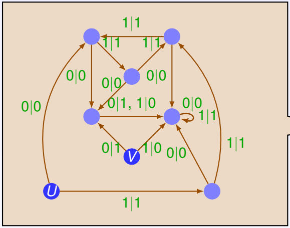
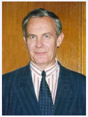
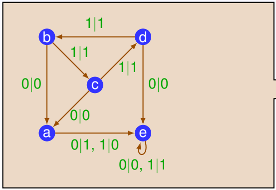


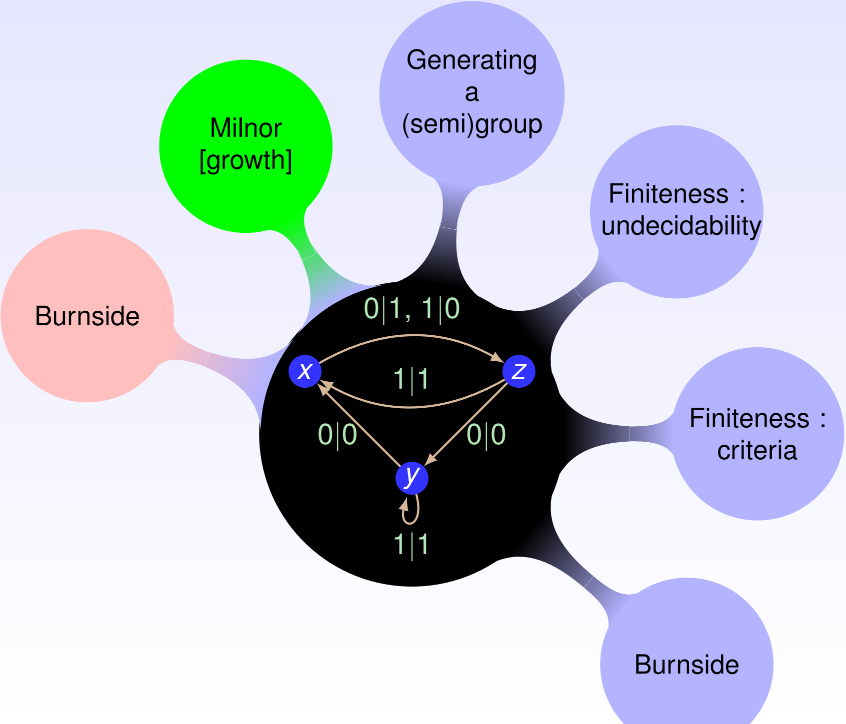


Aleshin

1980



Grigorchuk



Milnor [growth]
Generating a (semi)group
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Burnside
x
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0|1, 1|0
1|1
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Growth

$$\mathbb{Z}^2 = \langle a = (0,1), b = (1,0) \mid ab = ba \rangle$$

Growth

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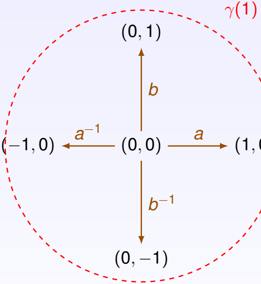
$$\gamma(0) = 1$$

$(0,0)$

Growth

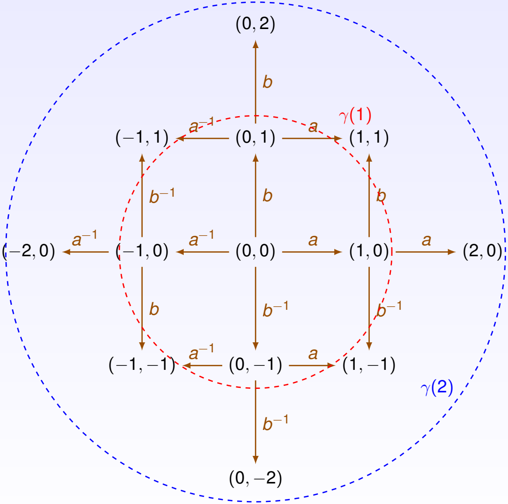
$$\mathbb{Z}^2 = \langle a = (0,1), b = (1,0) \mid ab = ba \rangle$$



$$\gamma(0) = 1$$
$$\gamma(1) = 5$$

Growth

$\mathbb{Z}^2 = \langle a = (0,1), b = (1,0) \mid ab = ba \rangle$



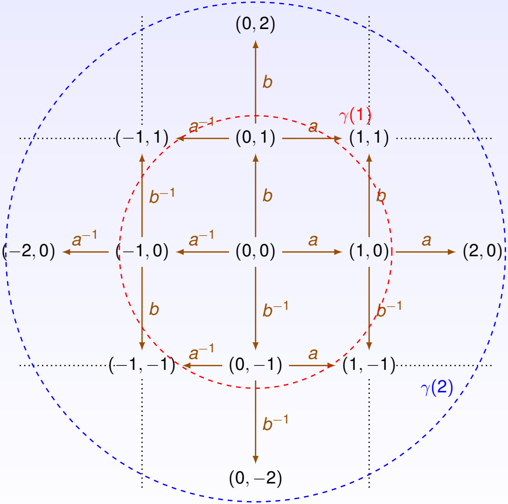
$\gamma(0) = 1$

$\gamma(1) = 5$

$\gamma(2) = 13$

Growth

$$\mathbb{Z}^2 = \langle a = (0,1), b = (1,0) \mid ab = ba \rangle$$



$\gamma(0) = 1$

$\gamma(1) = 5$

$\gamma(2) = 13$

$\vdots$

$\gamma(n) = 2n^2 + 2n + 1$

Growth

$$\mathbb{F}_2 = \langle a, b \rangle$$

Growth

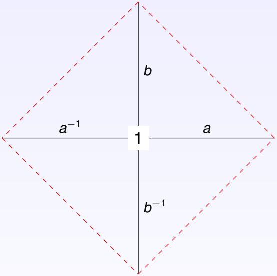
$$\mathbb{F}_2 = \langle a, b \rangle$$

$$\gamma(0) = 1$$

1

Growth

$\mathbb{F}_2 = \langle a, b \rangle$



$\gamma(0) = 1$
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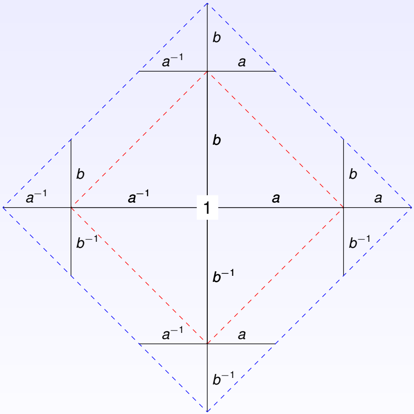
Growth

$\mathbb{F}_2 = \langle a, b \rangle$



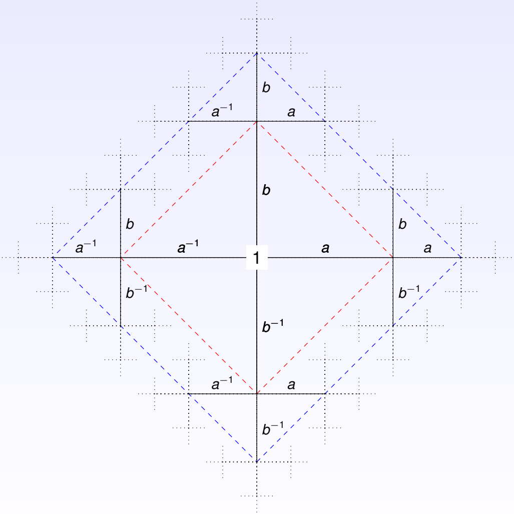
$\gamma(0) = 1$

$\gamma(1) = 5$

$\gamma(2) = 17$

Growth

$\mathbb{F}_2 = \langle a, b \rangle$



$\gamma(0) = 1$

$\gamma(1) = 5$

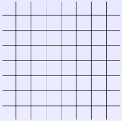
$\gamma(2) = 17$

$\vdots$

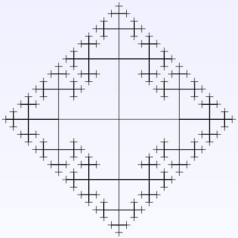
$\gamma(n) = 2 \times 3^n - 1$

bounded growth : finite groups

polynomial growth : $\mathbb{Z}^d$, abelian groups



exponential growth : $\mathbb{F}_d$



bounded growth : finite groups

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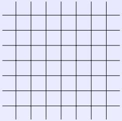
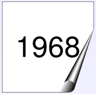
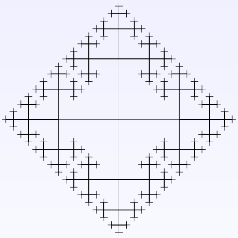




Is there an in between ?

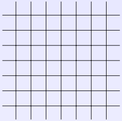
Milnor

exponential growth : $\mathbb{F}_d$



bounded growth : finite groups

polynomial growth : $\mathbb{Z}^d$, abelian groups

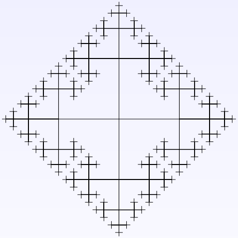


1968

Is there an in between ?

Milnor

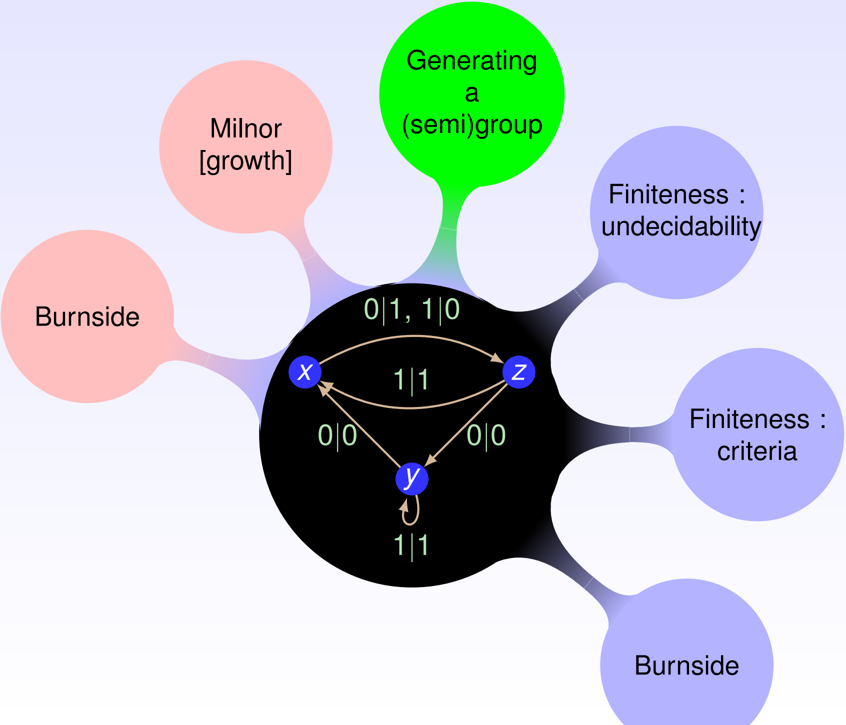
exponential growth : $\mathbb{F}_d$



1983

Yes!

Grigorchuk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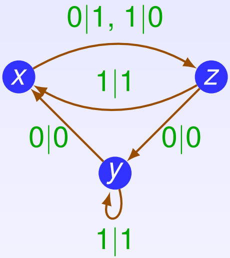
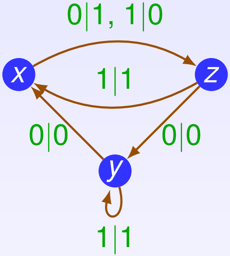
(Semi)Groups generated by automata



$\mathcal{A}$

(Semi)Groups generated by automata

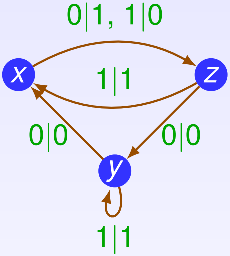
$\mathcal{A}$



ρ_x : $01000 \mapsto 11100$

(Semi)Groups generated by automata

$\mathcal{A}$

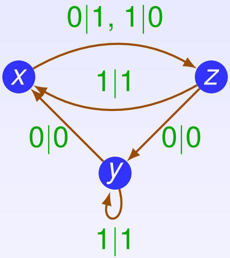


ρ_x : $\quad 01000 \mapsto 11100$

$\qquad \Sigma^* \to \Sigma^*$

(Semi)Groups generated by automata

$\mathcal{A}$

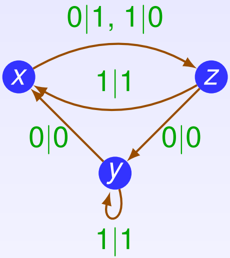


$\langle \mathcal{A} \rangle_+$ semigroup

- deterministic [fonctional]
- complete [defined all over Σ^*]

$$\rho_x : \quad 01000 \mapsto 11100$$
$$\Sigma^* \to \Sigma^*$$

(Semi)Groups generated by automata



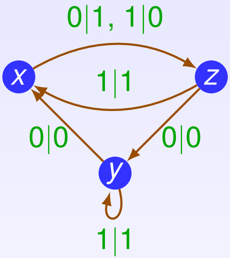
$\mathcal{A}$

$\langle \mathcal{A} \rangle_+$ semigroup
$\langle \mathcal{A} \rangle$ group

ρ_x : 01000 $\mapsto$ 11100
$\Sigma^* \rightarrow \Sigma^*$

- deterministic [fonctional]
- complete [defined all over Σ^*]
- the states permute the alphabet [for groups]

(Semi)Groups generated by automata

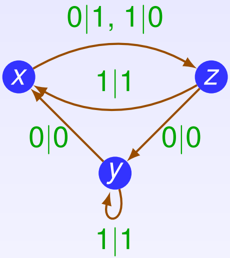


$\mathcal{A}$

$\langle \mathcal{A} \rangle_+$ semigroup
$\langle \mathcal{A} \rangle$ group

- acts on finite words
  $\rho_x : 01000 \mapsto 11100$

(Semi)Groups generated by automata



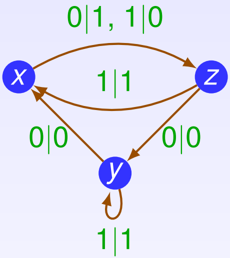
$\mathcal{A}$

$\langle \mathcal{A} \rangle_+$ semigroup
$\langle \mathcal{A} \rangle$ group

- acts on finite words
  ρ_x : $01000 \mapsto 11100$
- acts on infinite words
  ρ_x : $01000^\omega \mapsto 11(100)^\omega$

(Semi)Groups generated by automata

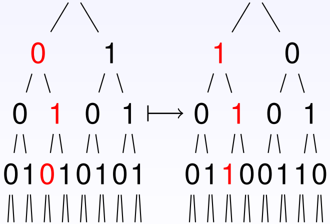
$\mathcal{A}$



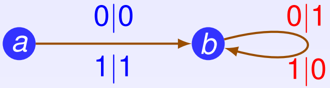
$\langle \mathcal{A} \rangle_+$ semigroup
$\langle \mathcal{A} \rangle$ group

- acts on finite words
  $\rho_x : 01000 \mapsto 11100$
- acts on infinite words
  $\rho_x : 01000^\omega \mapsto 11(100)^\omega$
- acts on the regular rooted tree

ρ_x :



Some examples



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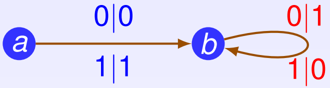
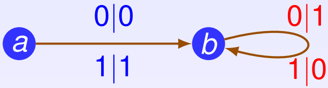


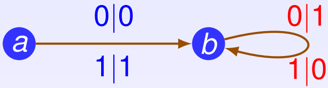
$$\rho_b : \begin{array}{cc} 001110010101\ldots & 101110010101\ldots \\ \downarrow & \downarrow \\ 110001101010\ldots & 010001101010\ldots \end{array}$$

Some examples



$$\rho_b : \begin{array}{cc} 001110010101\ldots & 101110010101\ldots \\ \downarrow & \downarrow \\ 110001101010\ldots & 010001101010\ldots \end{array} \qquad \rho_b^2 = \mathrm{id}_{\Sigma^*}$$

Some examples

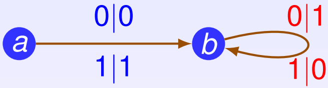


ρ_a : 001110010101... ↧ 010001101010... 101110010101... ↧ 110001101010...

ρ_b : 001110010101... ↧ 110001101010... 101110010101... ↧ 010001101010...

$\rho_b^2 = \mathrm{id}_{\Sigma^*}$

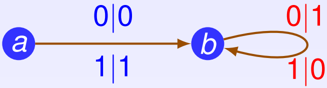
Some examples



$$\rho_a : \begin{array}{cc} 001110010101\ldots & 101110010101\ldots \\ \downarrow & \downarrow \\ 010001101010\ldots & 110001101010\ldots \end{array} \qquad \rho_a^2 = \mathrm{id}_{\Sigma^*}$$

$$\rho_b : \begin{array}{cc} 001110010101\ldots & 101110010101\ldots \\ \downarrow & \downarrow \\ 110001101010\ldots & 010001101010\ldots \end{array} \qquad \rho_b^2 = \mathrm{id}_{\Sigma^*}$$

Some examples

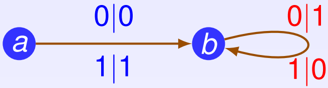


$$\rho_a : \quad 001110010101\ldots \mapsto 010001101010\ldots \qquad 101110010101\ldots \mapsto 110001101010\ldots \qquad \rho_a^2 = \mathrm{id}_{\Sigma^*}$$

$$\rho_b : \quad 001110010101\ldots \mapsto 110001101010\ldots \qquad 101110010101\ldots \mapsto 010001101010\ldots \qquad \rho_b^2 = \mathrm{id}_{\Sigma^*}$$

$$\rho_b\rho_a = \rho_a\rho_b : \quad 001110010101\ldots \mapsto 101110010101\ldots \qquad 101110010101\ldots \mapsto 001110010101\ldots$$

Some examples

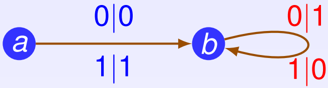


$$\rho_a : \begin{array}{ccc} 001110010101\ldots & & 101110010101\ldots \\ \downarrow & & \downarrow \\ 010001101010\ldots & & 110001101010\ldots \end{array} \qquad \rho_a^2 = \mathrm{id}_{\Sigma^*}$$

$$\rho_b : \begin{array}{ccc} 001110010101\ldots & & 101110010101\ldots \\ \downarrow & & \downarrow \\ 110001101010\ldots & & 010001101010\ldots \end{array} \qquad \rho_b^2 = \mathrm{id}_{\Sigma^*}$$

$$\rho_b\rho_a = \rho_a\rho_b : \begin{array}{ccc} 001110010101\ldots & & 101110010101\ldots \\ \downarrow & & \downarrow \\ 101110010101\ldots & & 001110010101\ldots \end{array} \qquad (\rho_a\rho_b)^2 = \mathrm{id}_{\Sigma^*}$$

Some examples



$\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}$

$$\rho_a : \begin{array}{cc} 001110010101\ldots & 101110010101\ldots \\ \downarrow & \downarrow \\ 010001101010\ldots & 110001101010\ldots \end{array} \qquad \rho_a^2 = \mathrm{id}_{\Sigma^*}$$

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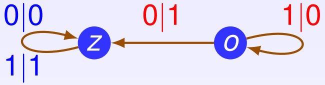
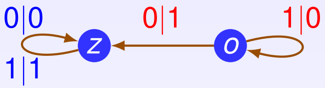


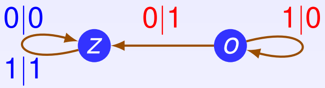
Some examples



z : 111001010100 $\mapsto$ 111001010100

o : 111001010100 $\mapsto$ 000101010100

Some examples



$z : 111001010100 \mapsto 111001010100$

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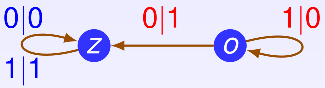
Some examples



$z : 111001010100 \mapsto 111001010100 \qquad + 0$

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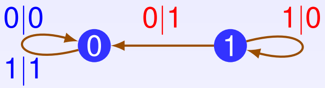
Some examples



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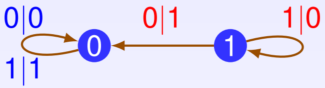

$\mathbb{N}, \mathbb{Z}$

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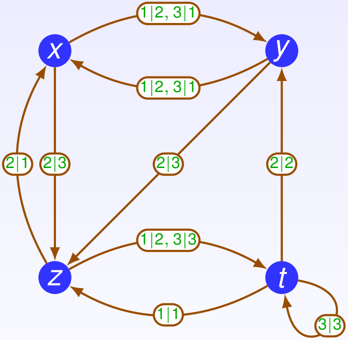
How automata can contribute

The word problem



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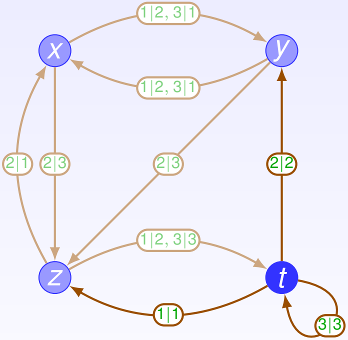

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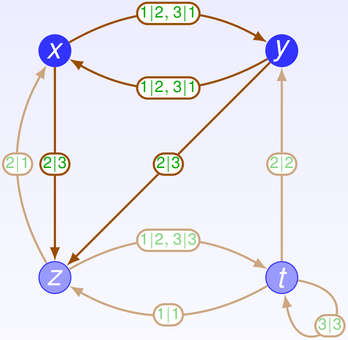

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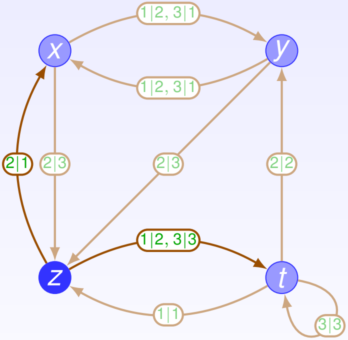

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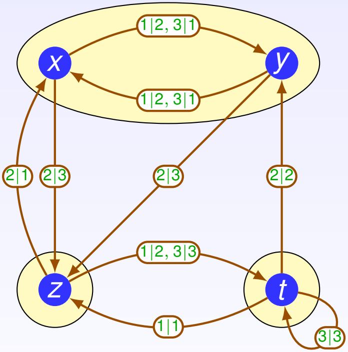

How automata can contribute

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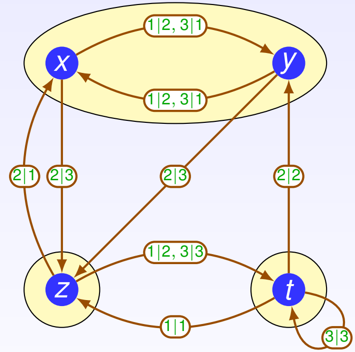


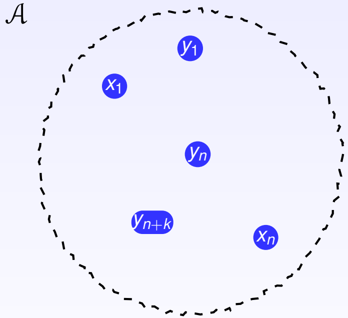
+ classical minimisation :

$$[x] = [y] \iff \rho_{x_{|\Sigma^*}} = \rho_{y_{|\Sigma^*}}$$

How automata can contribute

The word problem

$$\rho_{x_1 \cdots x_n} \stackrel{?}{=} \rho_{y_1 \cdots y_{n+k}}$$



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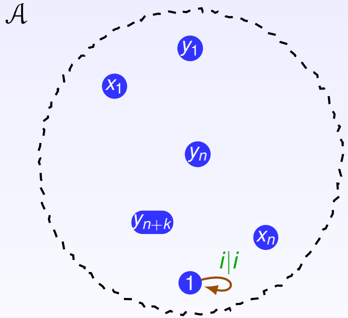

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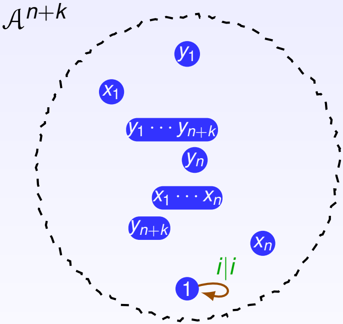

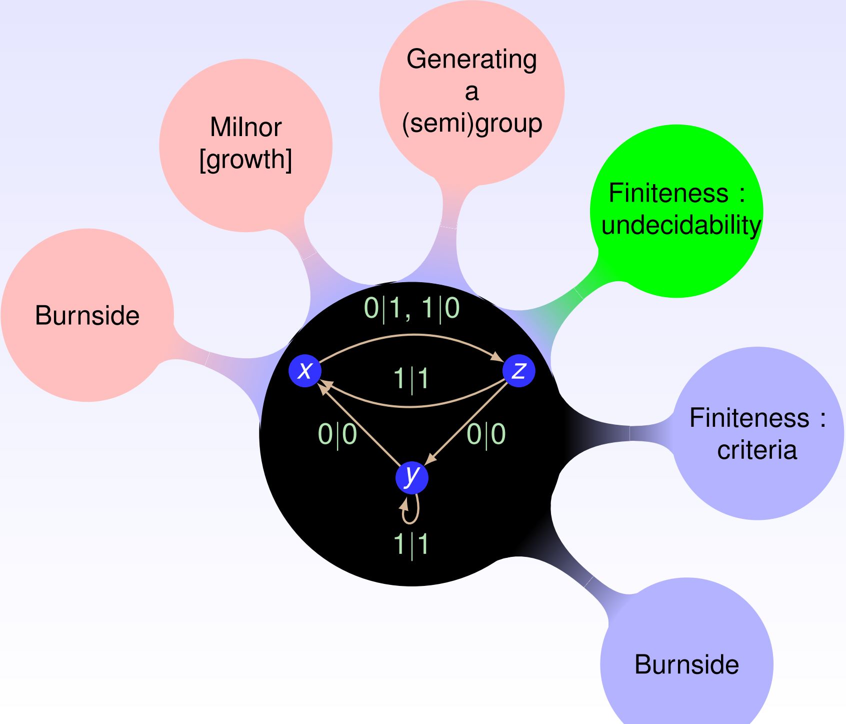
Generating a (semi)group
Milnor [growth]
Finiteness : undecidability
Burnside
0|1, 1|0
x
z
1|1
0|0
0|0
y
1|1
Finiteness : criteria
Burnside

Finiteness of automaton (semi)groups

Theorem [Gillibert'13]

The finiteness of an automaton semigroup is undecidable.

A lot of partial results

- families of automata where the finiteness is decidable
- sufficient or necessary conditions of finiteness

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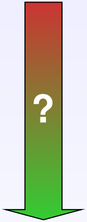

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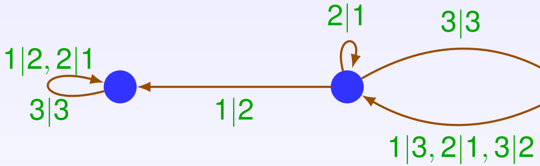

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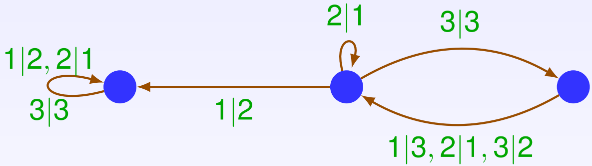

$$1\,494\,186\,269\,970\,473\,680\,896 = 2^{64} \cdot 3^{4} \approx 1.5 \times 10^{21}$$

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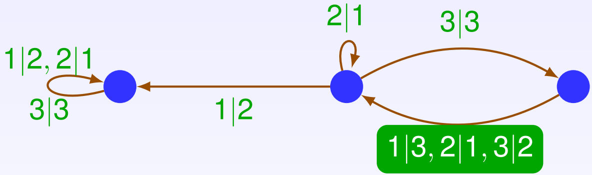

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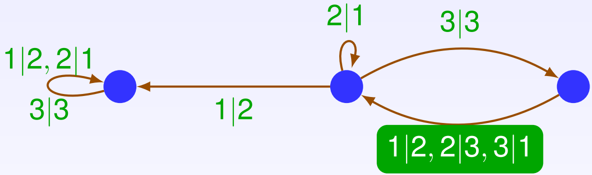

infinite

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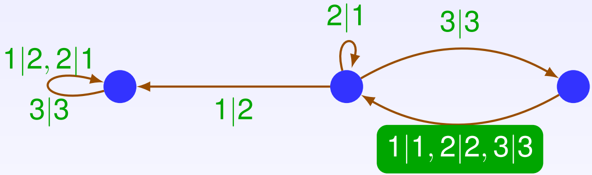

256

A lot of partial results

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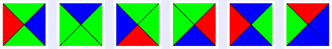
General case : finiteness is undecidable [Gillibert'13]

Wang's tile : 

General case : finiteness is undecidable [Gillibert'13]

Wang's tiles : 

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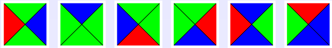
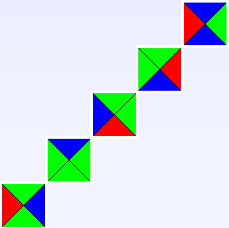
Wang's tiles : 

tiling



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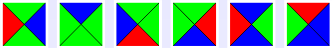
Wang's tiles : 

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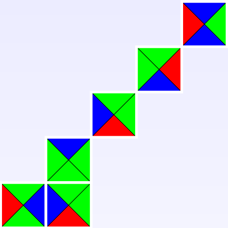

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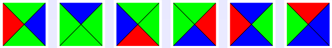
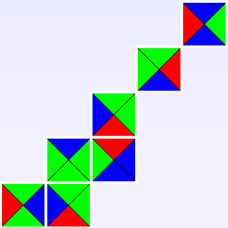
Wang's tiles : 

tiling 

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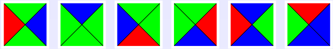
Wang's tiles : 

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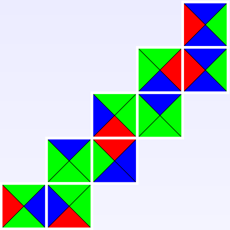

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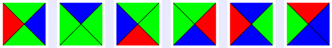
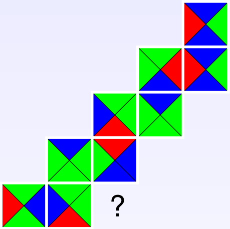
Wang's tiles : 

NW-deterministic tiling



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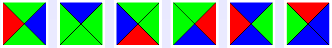
Wang's tiles : 

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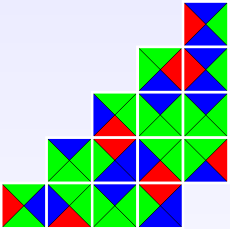

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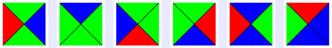
Wang's tiles : 

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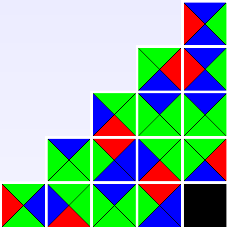

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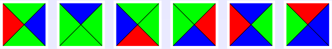
Wang's tiles : 

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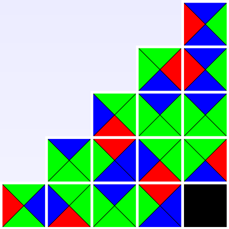
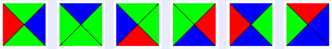


[Kari'92]

It is undecidable if a NW-deterministic tiling has a valid tiling for the plane.

General case : finiteness is undecidable [Gillibert'13]

Wang's tiles :  = T

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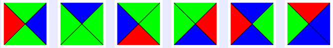
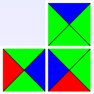
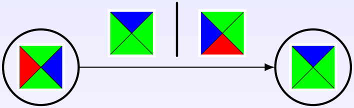
Wang's tiles :  = T

$\mathcal{A}_T$: 



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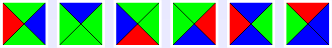
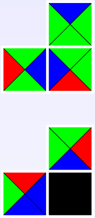
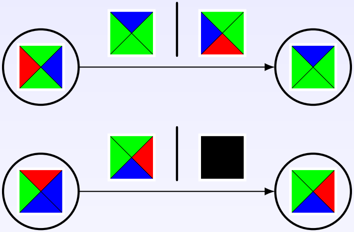
Wang's tiles :  = T

$\mathcal{A}_T$: 



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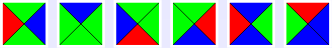
Wang's tiles :  $= T$

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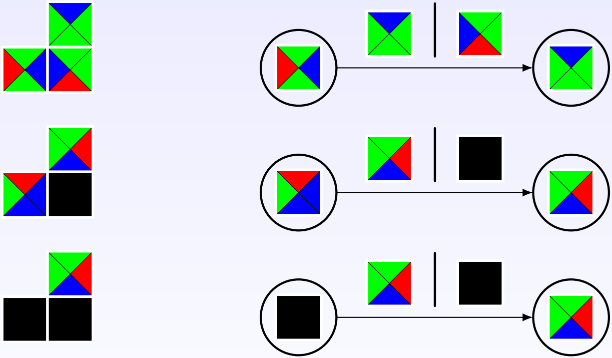

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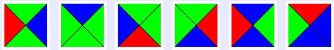
Wang's tiles :  = T

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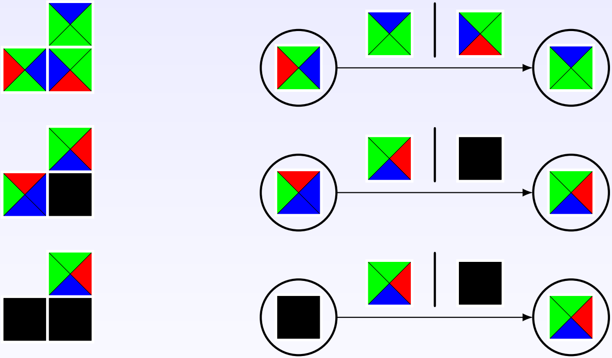
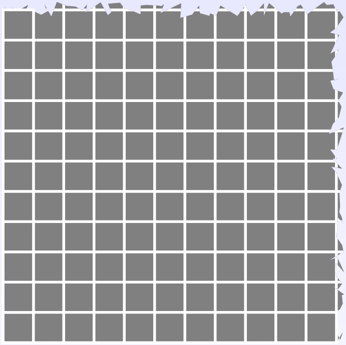


[Gillibert'13]

T has a valid Wang tiling for the plane $\Longleftrightarrow \langle \mathcal{A}_T \rangle_+$ is infinite



T has a valid tiling for the plane

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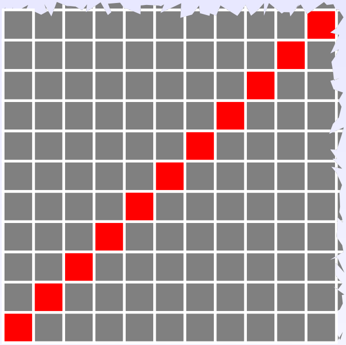

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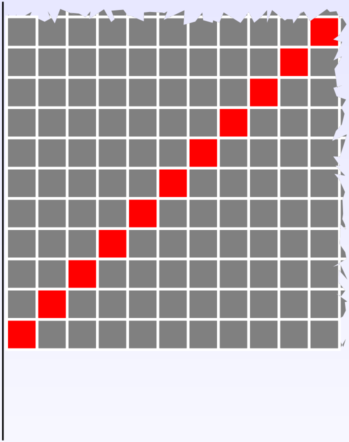

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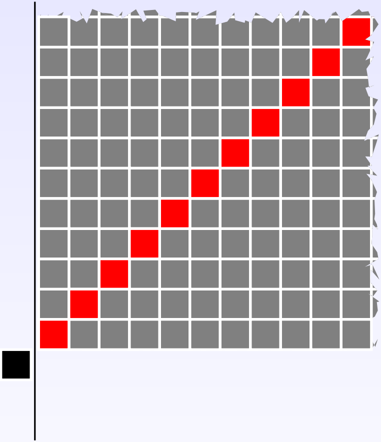

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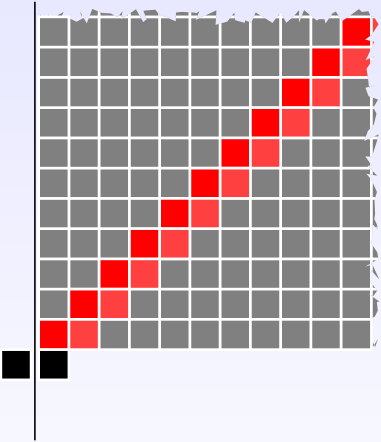

T has a valid tiling for the plane

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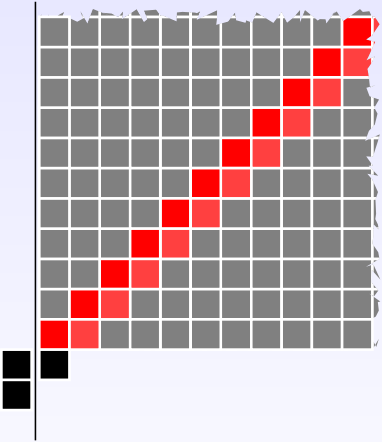

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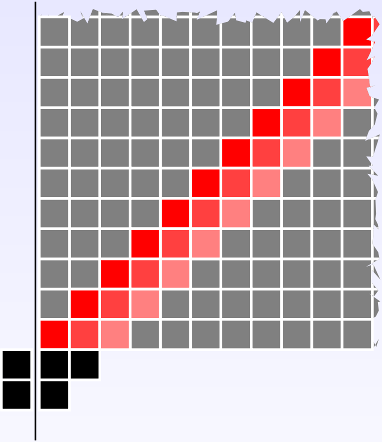

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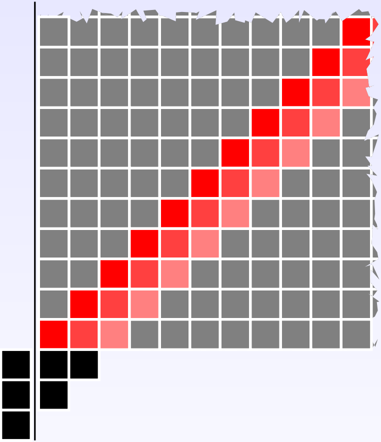

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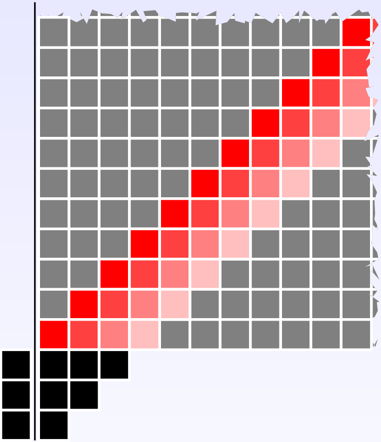

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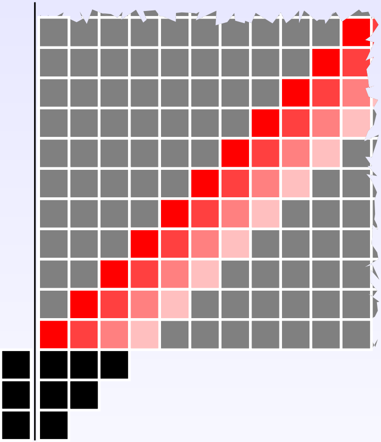


T has a valid tiling for the plane

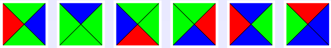
$\Downarrow$

the action of ■ has infinite order

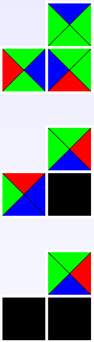
[Gillibert'13]

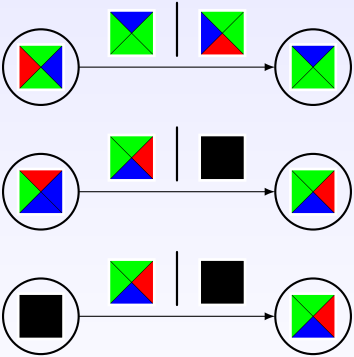
T has a valid Wang tiling for the plane $\Longleftrightarrow \langle \mathcal{A}_T \rangle_+$ is infinite

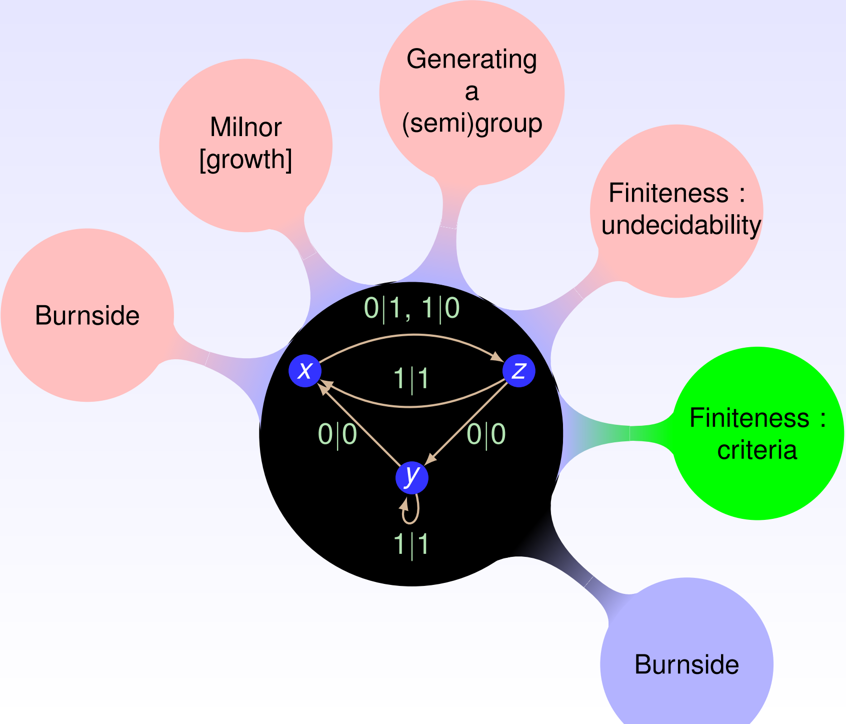
General case : finiteness is undecidable [Gillibert'13]

Wang's tiles :  = T

$\mathcal{A}_T$:







Generating a (semi)group
Milnor [growth]
Finiteness : undecidability
Burnside
0|1, 1|0
x
z
1|1
0|0
0|0
y
1|1
Finiteness : criteria
Burnside

Finiteness of automaton (semi)groups

Theorem [Gillibert'13]

The finiteness of an automaton semigroup is undecidable.

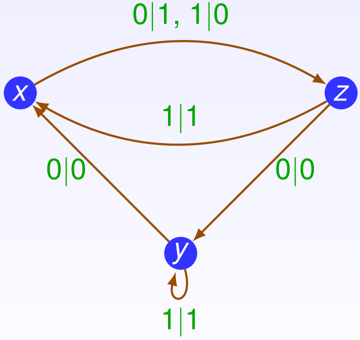
A lot of partial results

- families of automata where the finiteness is decidable
- sufficient or necessary conditions of finiteness

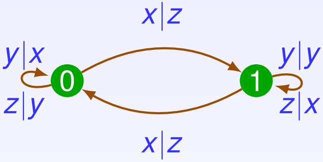
The dual automaton

inverting the letters and the states roles

$\mathcal{A}$



$\mathfrak{d}(\mathcal{A})$



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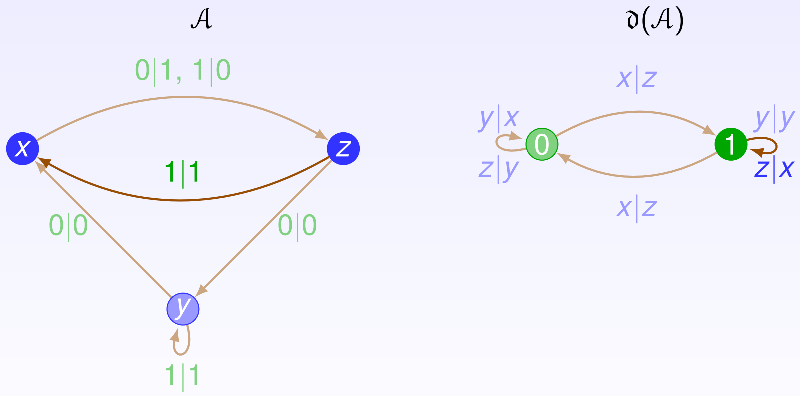

The dual automaton

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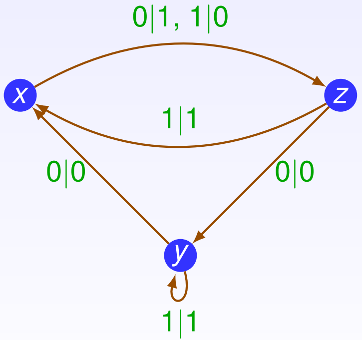

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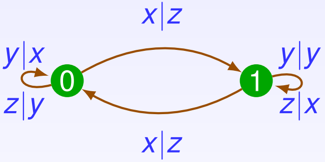

$\langle \mathfrak{d}(\mathcal{A}) \rangle_+$ finite $\iff$ $\langle \mathcal{A} \rangle_+$ finite

$\langle \mathfrak{d}(\mathcal{A}) \rangle_+$: words acting on the states of $\mathcal{A}^n$

The dual automaton

δ_i : productions of $\mathfrak{d}(\mathcal{A})$

$\langle \mathfrak{d}(\mathcal{A}) \rangle_+$ finite $\Longrightarrow$ $\langle \mathcal{A} \rangle_+$ finite

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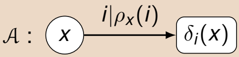

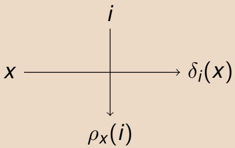

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$$\mathbf{w} \xrightarrow[\rho_{\mathbf{w}}(u_1)]{u_1} \delta_{u_1}(\mathbf{w}) \xrightarrow[\rho_{\delta_{u_1}(\mathbf{w})}(u_2)]{u_2} \cdots \qquad \cdots \to \delta_{u_1 u_2 \cdots u_{n-1}}(\mathbf{w}) \xrightarrow[\rho_{\delta_{u_1 \cdots u_{n-1}}(\mathbf{w})}(u_n)]{u_n}$$

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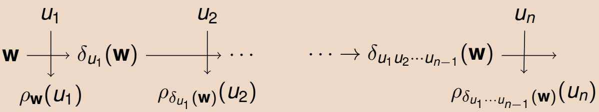

Cayley($\langle \mathfrak{d}(\mathcal{A}) \rangle_+$) :

$\langle \mathfrak{d}(\mathcal{A}) \rangle_+$ finite $\Longrightarrow$ $\langle \mathcal{A} \rangle_+$ finite

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The dual automaton

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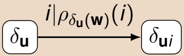

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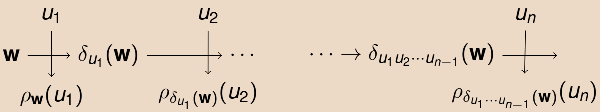

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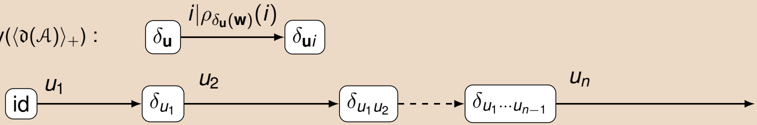

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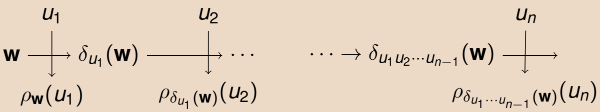

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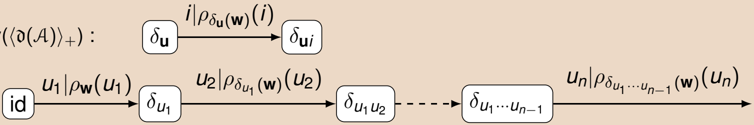

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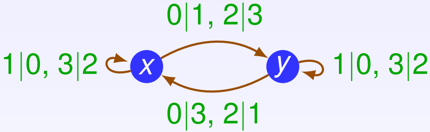
$\mathfrak{md}$-triviality

Theorem [Akhavi K Lombardy Mairesse Picantin'12]

$$\mathcal{A}\ \mathfrak{md}\text{-trivial} \Longrightarrow \langle \mathcal{A} \rangle_+ \text{ finite}$$



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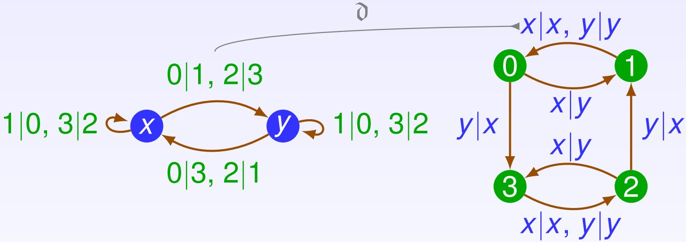

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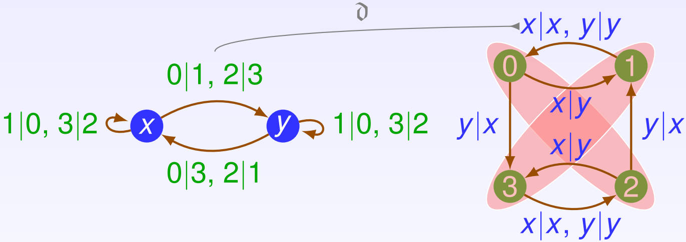

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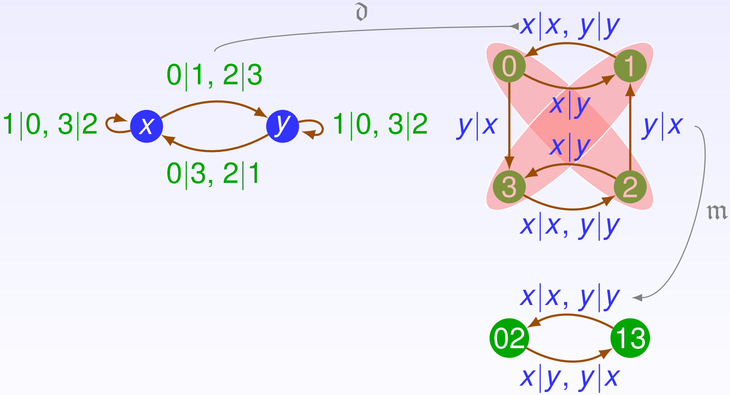

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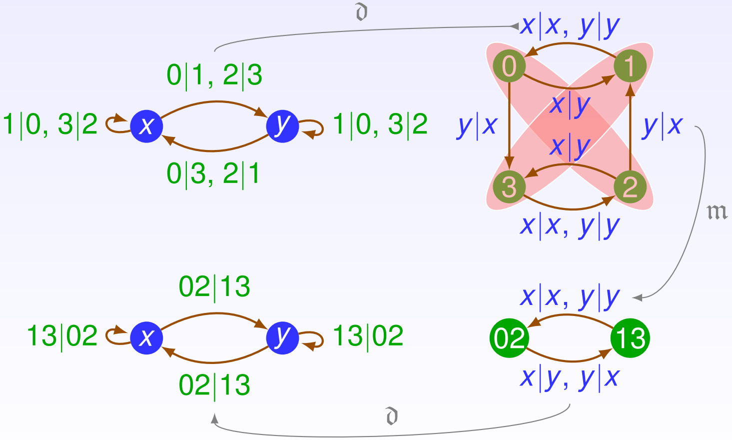

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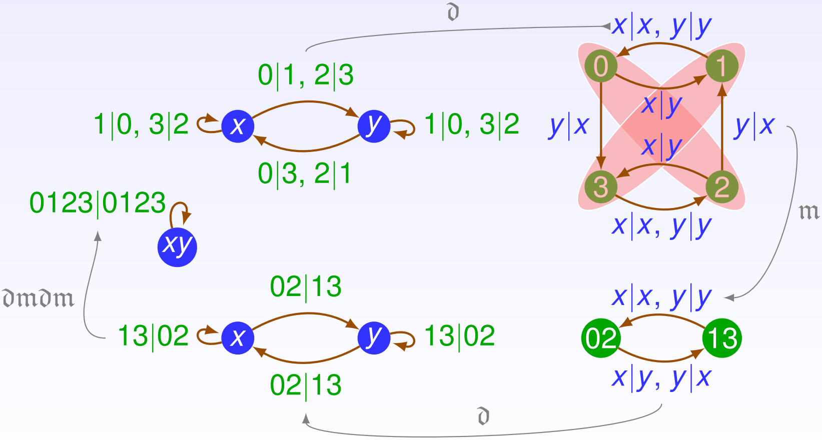


$\mathfrak{md}$-triviality

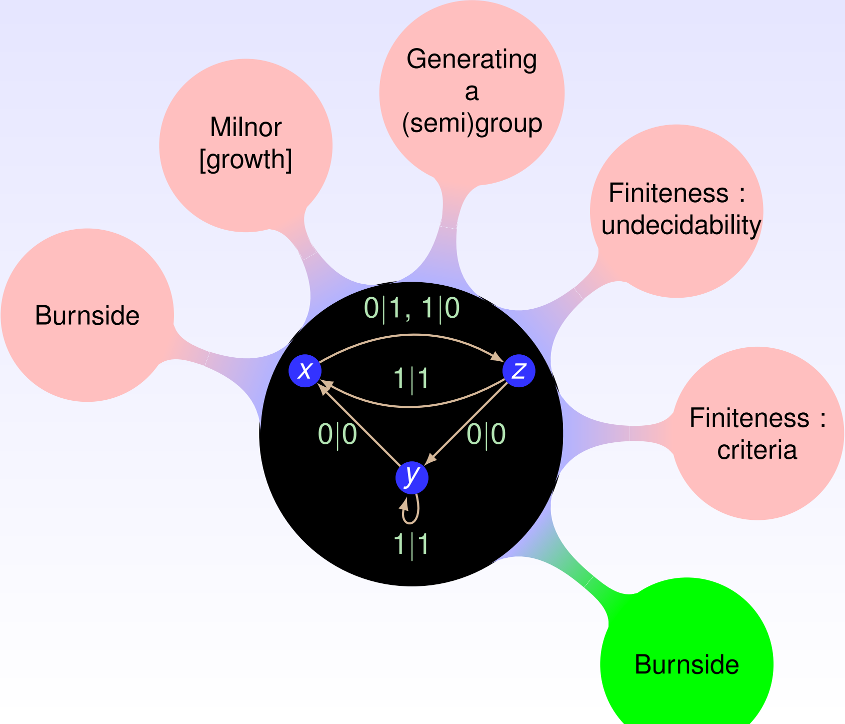
[Akhavi K Lombardy Mairesse Picantin'12]

$\mathcal{A}$ $\mathfrak{md}$-trivial $\not\Leftarrow$ $\langle\mathcal{A}\rangle_+$ finite

$\mathfrak{md}$-triviality

Theorem [K, STACS'13]

$\mathcal{A}$ 2-state inv.-rev. : $\mathcal{A}$ $\mathfrak{md}$-trivial $\Longleftarrow$ $\langle\mathcal{A}\rangle$ finite


Generating a (semi)group
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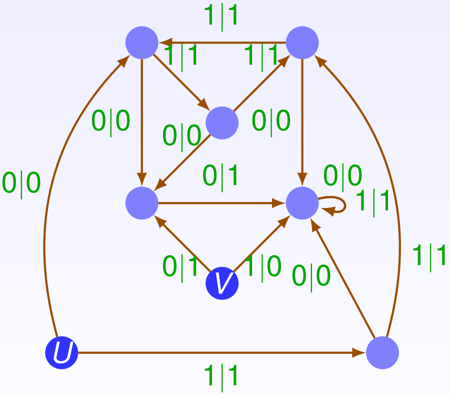
The Burnside problem

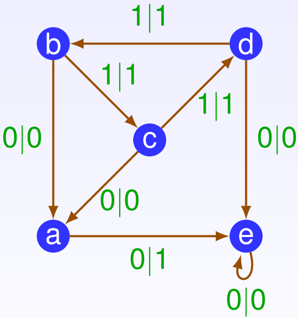
Is a finitely generated group whose all elements have finite order necessarily finite ?

The Burnside problem

A common point between these examples ?

Is a finitely generated group whose all elements have finite order necessarily finite ?





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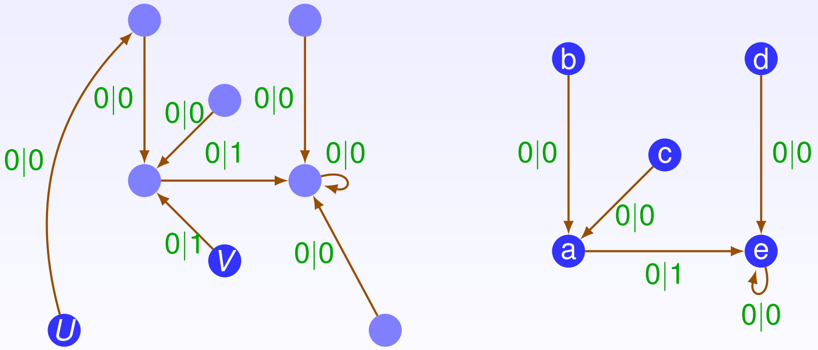

The Burnside problem and the reversibility

A common point between these examples ?

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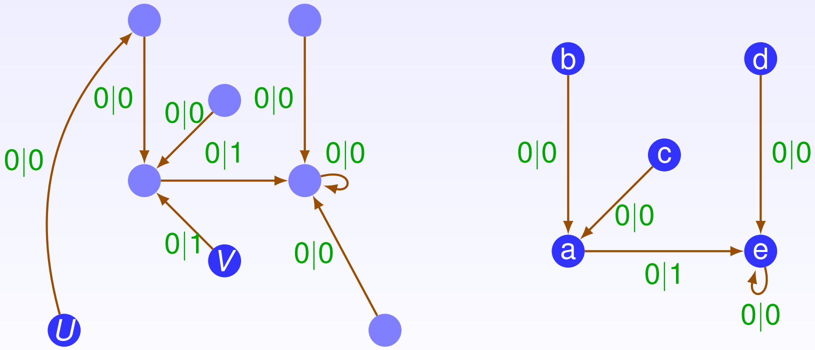


not reversible !

(letters are not permutations of the states)

The Burnside problem and the reversibility

Natural question

Can a reversible automaton generate an infinite Burnside group ?

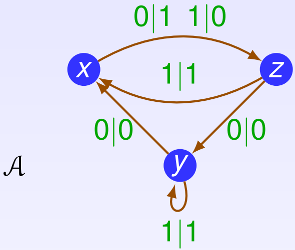
The Burnside problem and the reversibility

Natural question

Can a reversible automaton generate an infinite Burnside group ?

impossible :

- 2-state [K., STACS'13]
- connected 3-state [K. Picantin Savchuk'15]
- no bireversible component [Godin K. Picantin, LATA'15]



$$\rho_{x_i} : \Sigma^* \to \Sigma^*$$

$$\rho_{x_1 x_2 \dots x_n} = \rho_{x_n} \circ \cdots \circ \rho_{x_1}$$

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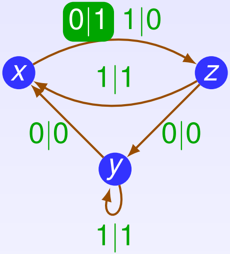
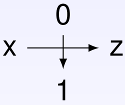
$\mathcal{A}$





x

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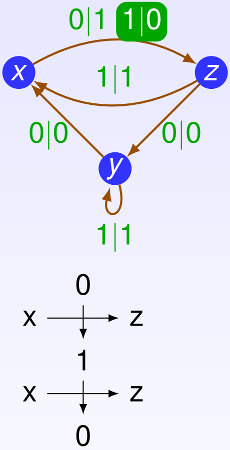

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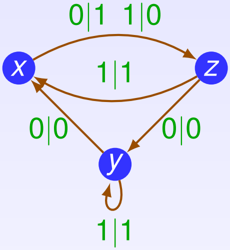

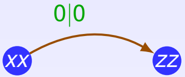

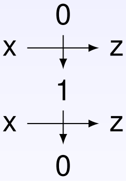

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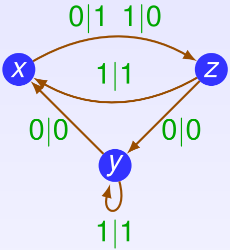

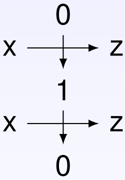

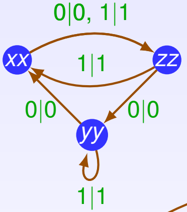

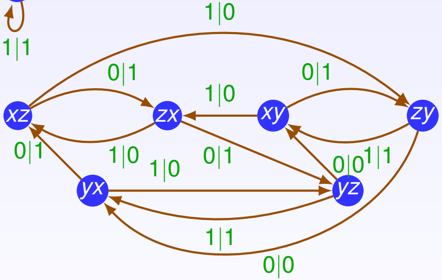

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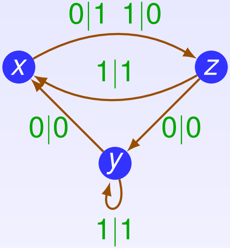

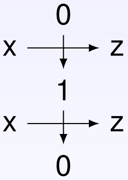

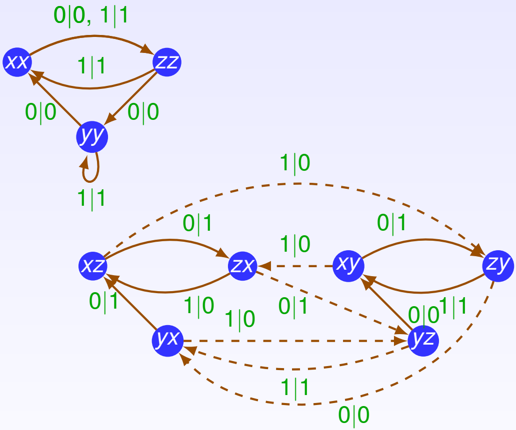

The connected components of the powers

$\mathcal{A}$ reversible (letters induce permutations of the stateset)

Finiteness problem

$\langle \mathcal{A} \rangle$ is finite $\Longleftrightarrow$ the cc of the $\mathcal{A}^n$ are bounded

Order problem

$\rho_{\mathbf{u}}$ has finite order $\Longleftrightarrow$ the cc of the $\mathbf{u}^n$ are bounded

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expressed as path properties in the orbit tree

Order problem

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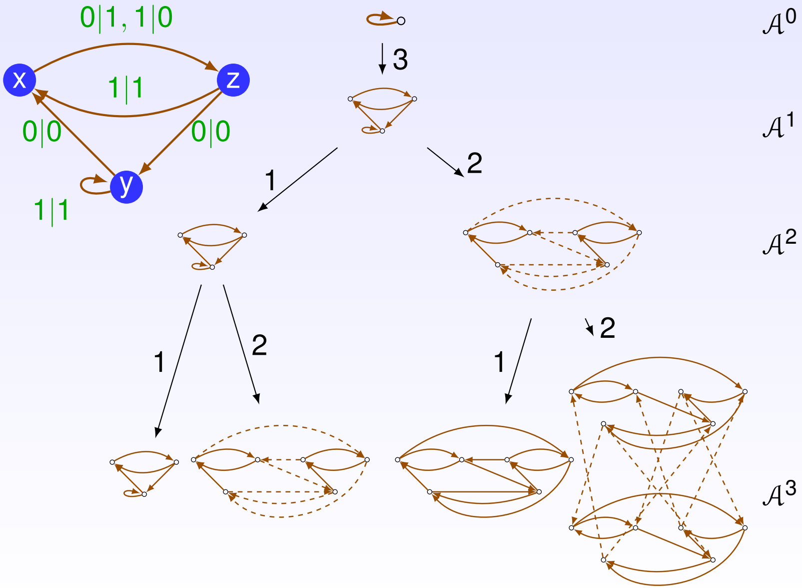
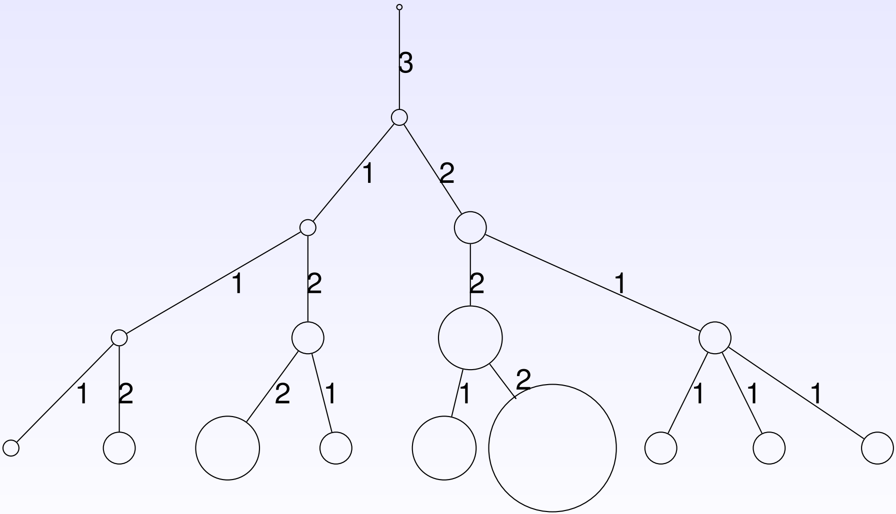
The labeled orbit tree

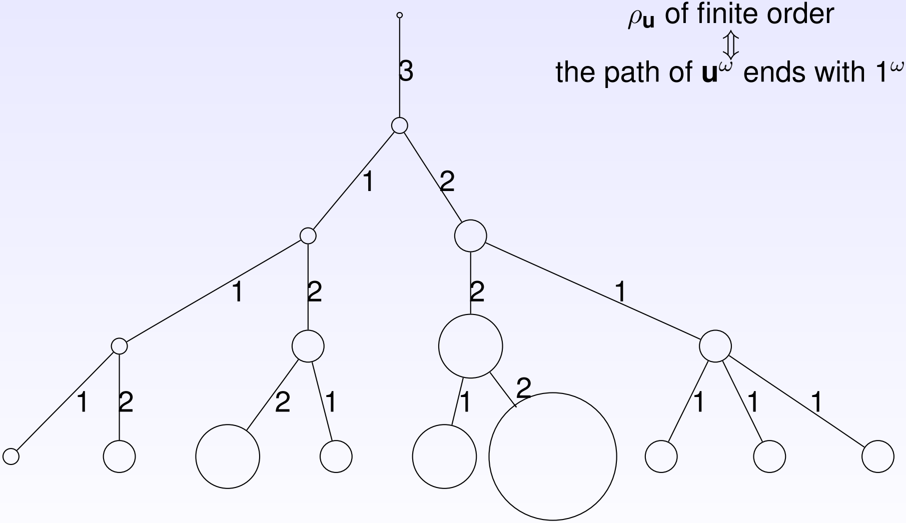


The labeled orbit tree



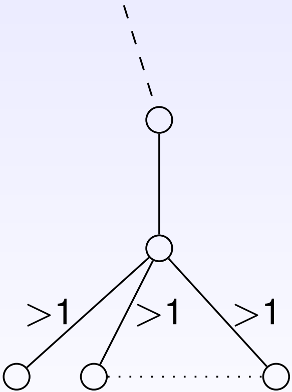
The labeled orbit tree

$\rho_{\mathbf{u}}$ of finite order
$\Updownarrow$
the path of $\mathbf{u}^\omega$ ends with 1^ω



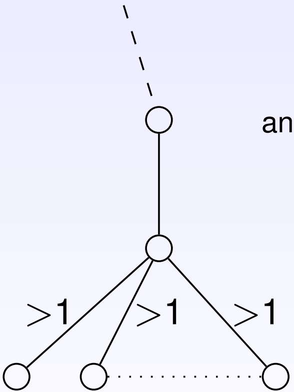
First result : no bireversible component

[Godin K. Picantin, LATA'15]



First result : no bireversible component

[Godin K. Picantin, LATA'15]

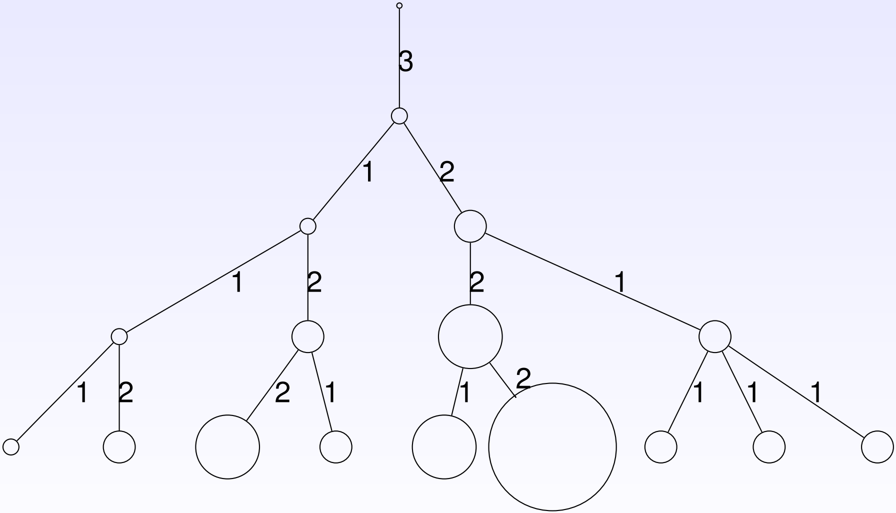


no path ends with 1^ω

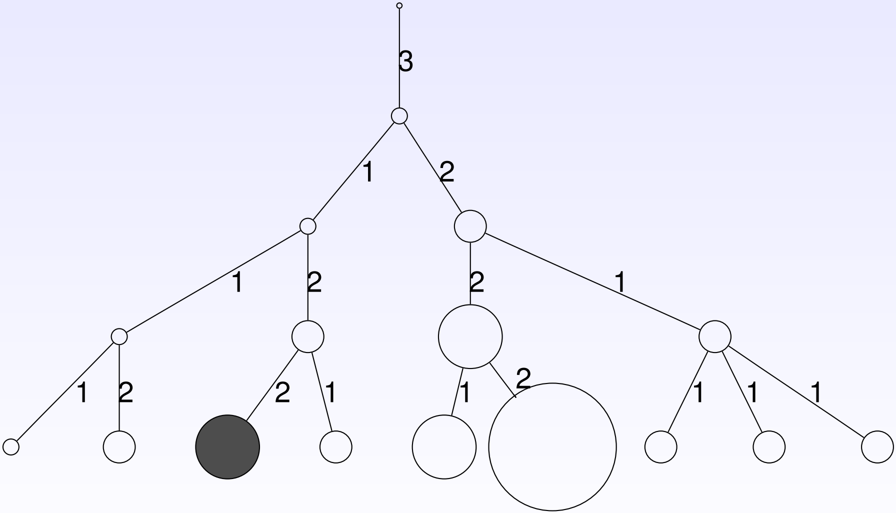
$\Downarrow$

any element of $\langle \mathcal{A} \rangle_+$ has infinite order

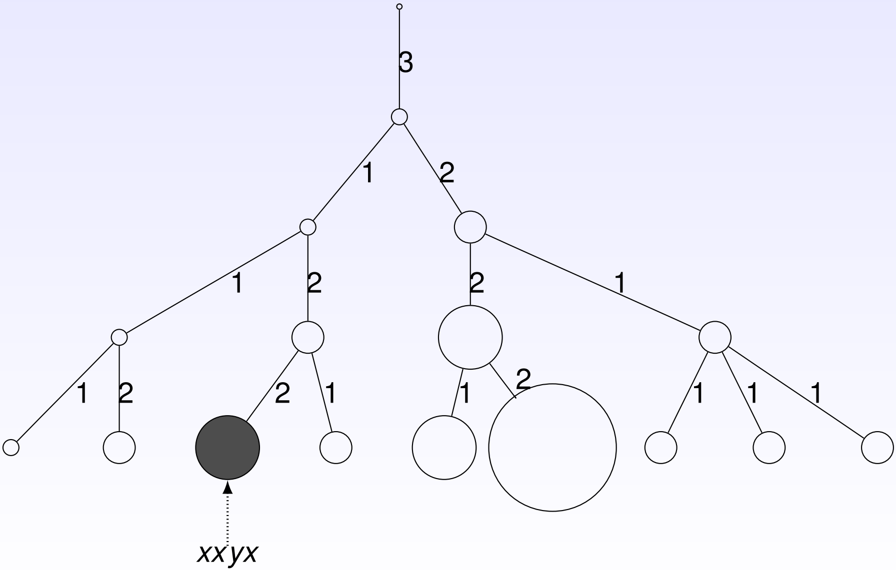
The labeled orbit tree



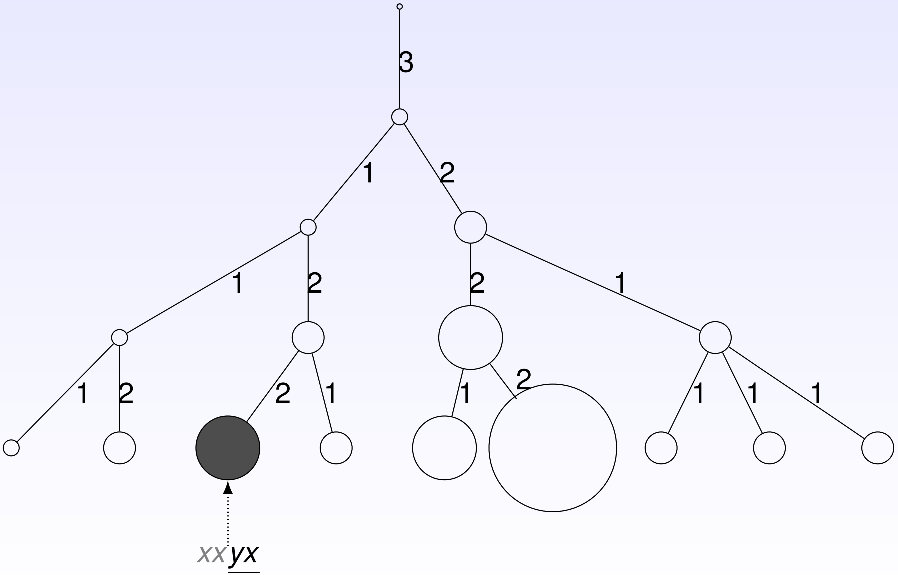
The labeled orbit tree



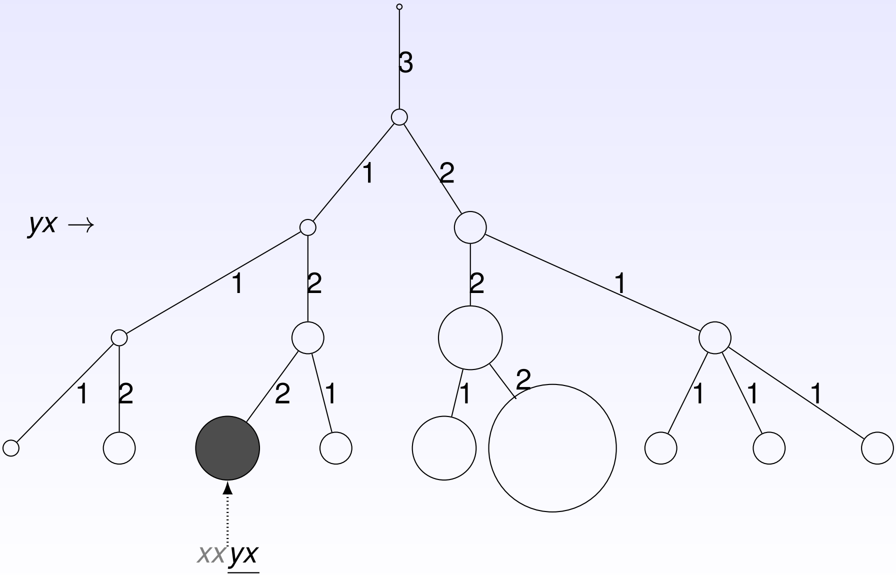
The labeled orbit tree



The labeled orbit tree



The labeled orbit tree



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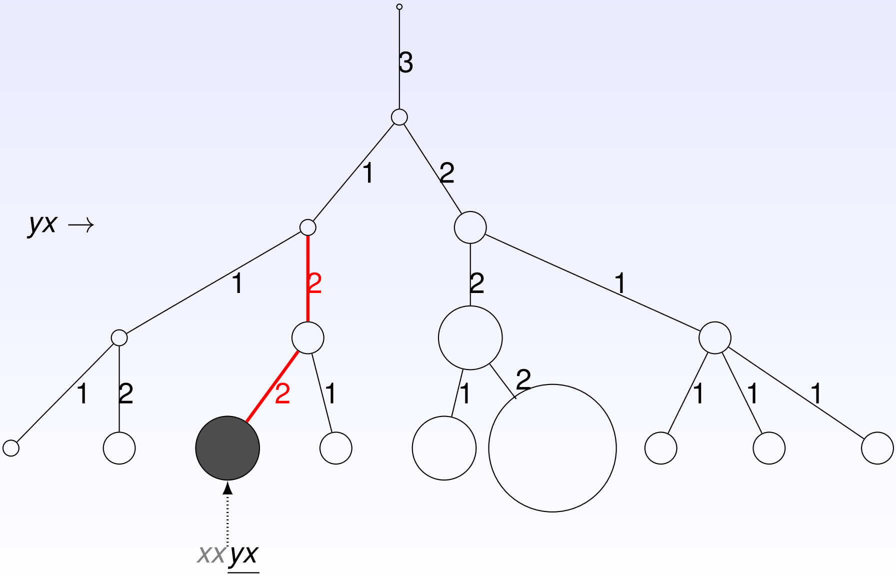
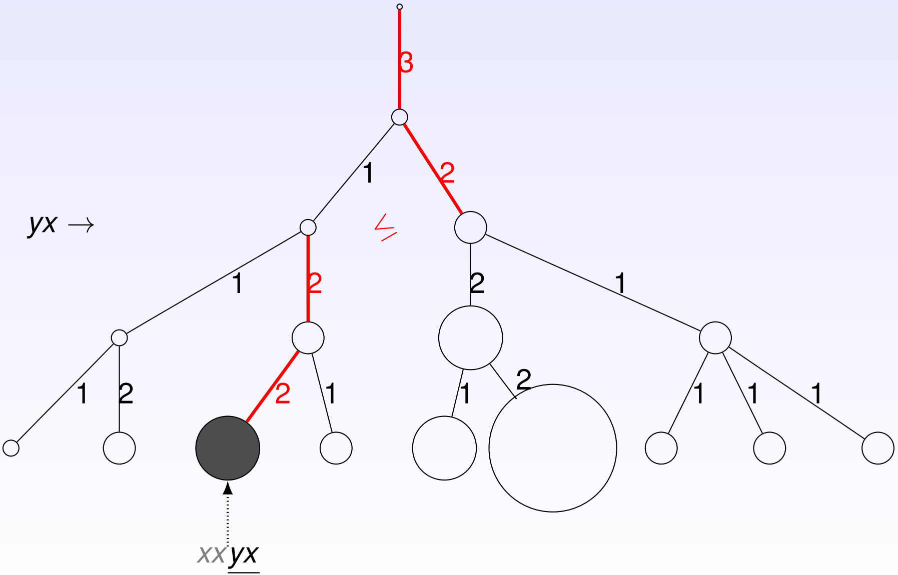
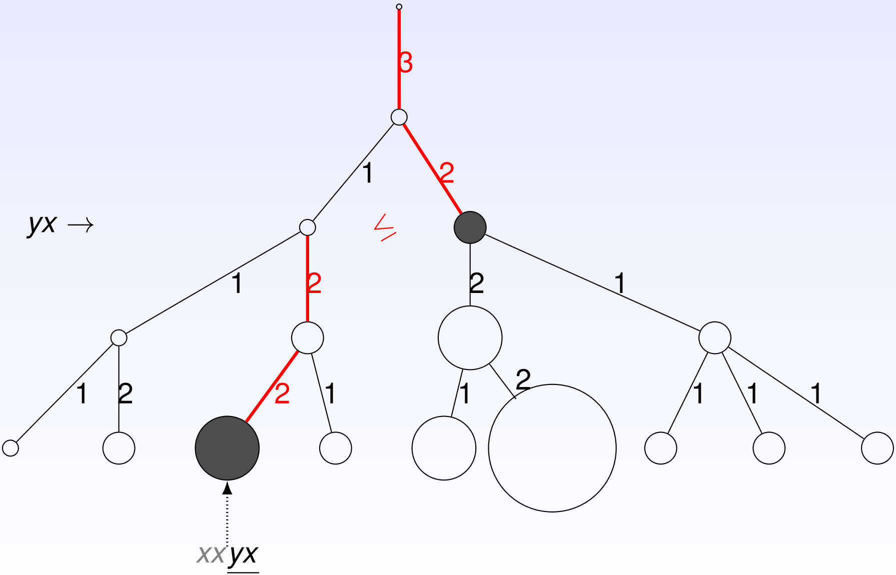


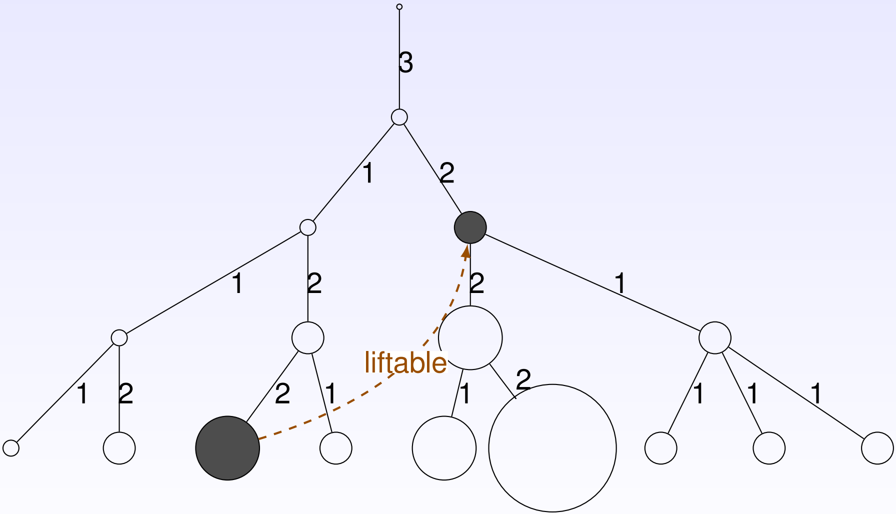
The labeled orbit tree



The labeled orbit tree

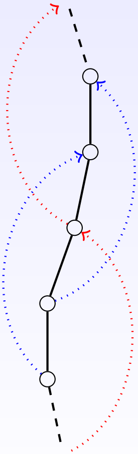


The labeled orbit tree



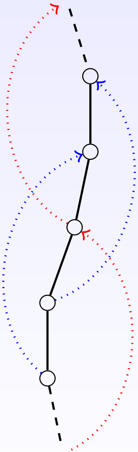
Self-liftable paths

k-self-liftable paths



Self-liftable paths

k-self-liftable paths



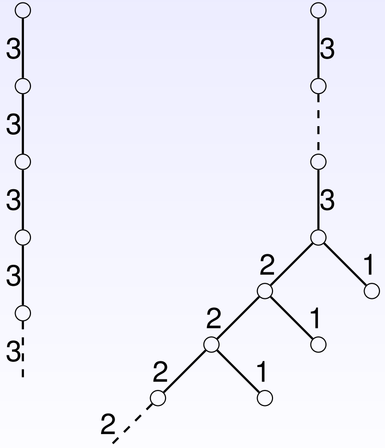
a path is self-liftable

$\Updownarrow$

it is the path of $\mathbf{u}^\omega$
(for some $\mathbf{u}$)

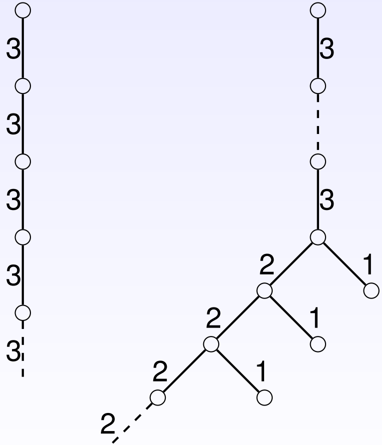
Second result : 3-state connected automata

[K. Picantin Savchuk'15]



Second result : 3-state connected automata

[K. Picantin Savchuk'15]



$\exists$ 1-self-liftable paths 3^ω or $3^n 2^\omega$

$\Downarrow$

$\exists$ elements of infinite order

Future work

Short term

- Can a reversible automaton generate an infinite Burnside group ?
- Is the finiteness of an automaton group decidable ?
- Is the order problem decidable in an automaton group ?

Long term

- Is the growth type of an automaton group decidable ?